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Comment on a Proposed Super-Kamiokande Test for Quantum Gravity Induced Decoherence Effects

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ABSTRACT

Lisi, Marrone, and Montanino have recently proposed a test for quantum gravity induced decoherence effects in neutrino oscillations observed at Super-Kamiokande. We comment here that their equations have the same qualitative form as the energy conserving objective state vector reduction equations discussed by a number of authors. However, using the Planckian parameter value proposed to explain state vector reduction leads to a neutrino oscillation effect many orders of magnitude smaller than would be detectable at Super-Kamiokande. Similar estimates hold for the Girardi, Rimini, and Weber spontaneous localization approach to state vector reduction, and our remarks are relevant as well to proposed K meson and B meson tests of gravity induced decoherence.

There has recently been considerable interest in testing for possible modifications in conventional quantum mechanics induced by Planck mass scale quantum fluctuations in the structure of spacetime. In an effective field theory approach, these are plausibly argued [1] to have the form of an extra "decoherence term" $\mathcal{D}[\rho]$ in the standard density matrix evolution equation, which becomes

$$\frac{d\rho}{dt} = -i[H,\rho] - \mathcal{D}[\rho] \quad , \tag{1}$$

where ρ is the density matrix, H is the Hamiltonian, and the decoherence term $\mathcal{D}$ has the dimensions of energy.

In many phenomenological applications, Eq. (1) is specialized by making several plausible assumptions about the structure of $\mathcal{D}$. First of all, the theory of open quantum systems suggests that Eq. (1) should correspond to the infinitesimal generator form of a *completely positive* map [2] on ρ, which requires that $\mathcal{D}$ should have the Lindblad form [3]

$$\mathcal{D}[\rho] = \sum_n [\{\rho, D_n^\dagger D_n\} - 2D_n \rho D_n^\dagger] \quad . \tag{2}$$

If one further requires the monotone increase of the von Neumann entropy $S = -\mathrm{Tr}\rho \log \rho$, and the conservation of energy, one adds the respective conditions that the "Lindblads" D_n should be self-adjoint, $D_n = D_n^\dagger$, and that they should commute with the Hamiltonian, $[D_n, H] = 0$. One then arrives at the form

$$\begin{aligned} \mathcal{D}[\rho] =& \sum_n [D_n, [D_n, \rho]] \quad , \\ [D_n, H] =& 0 \ , \quad \text{all} \ \ n \quad . \end{aligned} \tag{3}$$

Equation (3) is the starting point of an analysis recently given by Lisi, Marrone, and Montanino [4] of decoherence effects in the super-Kamiokande experiment, interpreted in terms of $\nu_\mu - \nu_\tau$ oscillations.

When specialized to a two-level quantum system, the only choices of D_n that commute with H are either $D_n = \kappa_n 1$, with 1 the unit operator, or $D_n = \lambda_n H$. The first choice is evidently trivial, since it makes no contribution to Eq. (3), and so can be ignored. Hence all terms in the sum over n in Eq. (3) have the same structure, corresponding to the second choice; defining $\lambda^2 = \sum_n \lambda_n^2$, it is no restriction to replace the sum on n in Eq. (3) with a single Lindblad $D = \lambda H$. In their analysis of the Super-Kamiokande data, Lisi et. al. define a parameter γ by

$$\gamma = 2\mathrm{Tr}\sum_n D_n^2 = 2\lambda^2 \mathrm{Tr} H^2 \quad , \tag{4}$$

and deduce the bound

$$\gamma < 3.5 \times 10^{-23}\mathrm{GeV} \quad . \tag{5}$$

Since in the two-level neutrino system we have $\mathrm{Tr}H^2 = \frac{1}{2}k^2$, with $k = \Delta m^2/(2E)$, where $\Delta m^2 = m_2^2 - m_1^2$ is the neutrino squared mass difference and E is the neutrino energy, the parameters λ and γ are related by

$$\lambda = \frac{\gamma^{\frac{1}{2}}}{k} = \frac{2E\gamma^{\frac{1}{2}}}{\Delta m^2} \quad . \tag{6}$$

Thus, using the Super-Kamiokande [5] value $\Delta m^2 = 3 \times 10^{-3}\mathrm{eV}^2$, and their maximum neutrino energy of $E \sim 10^3\mathrm{GeV}$, the bound of Eq. (5) on γ corresponds to a bound on λ of

$$\lambda < 4 \times 10^{12}\mathrm{GeV}^{-\frac{1}{2}} \quad . \tag{7}$$

The possibility that there may be decohering modifications to the Schrödinger equation, or to the corresponding density matrix evolution equation, has been extensively discussed over the past twenty years in the context of models for objective state vector reduction.

As surveyed by Adler and Horwitz [6], the form of the density matrix evolution assumed in these discussions is the Itô stochastic differential equation

$$d\rho = -i[H,\rho]dt - \frac{1}{8}\sigma^2[D,[D,\rho]]dt + \frac{1}{2}\sigma[\rho,[\rho,D]]dW_t \quad , \tag{8}$$

with D a Hermitian Lindblad operator driving the decoherence, and with dW_t an Itô stochastic differential obeying

$$dW_t^2 = dt \ , \quad dtdW_t = 0 \quad . \tag{9}$$

(One can readily generalize Eq. (8) to contain a sum over multiple Lindblads D_n, but this will not be needed in our analysis.) Two differing choices of the Lindblad D have been widely discussed in the literature. The first [7], due to Girardi, Rimini, and Weber, as extended by Diósi and Girardi, Pearle, and Rimini, takes D to be a localizing operator in coordinate space; we will discuss this case later on. The second [8], emphasized recently by Percival and Hughston, takes D to be the Hamiltonian H, and this is the case on which we shall focus. As shown by Adler and Horwitz, when D is taken to be the Hamiltonian, Eq. (8) can be proved, with no approximations, to lead to state vector reduction to energy eigenstates with the correct probabilities as given by the quantum mechanical Born rule. To account for the observed absence of macroscopic spatial superpositions, one has to invoke energy shifts associated with environmental interactions which differ for macroscopic objects at different spatial locations; whether this leads to an empirically viable model for state vector reduction is presently an open question.

Making the choice $D = H$ in Eq. (8), and taking the stochastic expectation, leads to an evolution equation for the stochastic expectation of the density matrix identical in form with Eqs. (1-3) used by Lisi et. al. in their Super-Kamiokande analysis. If one assumes a quantum gravitational origin for the stochastic terms in Eq. (7), then the natural estimate [9] for the parameter σ is $\sigma \sim M_{\rm Planck}^{-\frac{1}{2}}$, which since $\sigma^2/8 = \lambda^2$ corresponds to

$$\lambda \sim (8M_{\rm Planck})^{-\frac{1}{2}} \sim 10^{-10}{\rm GeV}^{-\frac{1}{2}} \quad , \tag{10}$$

more than twenty orders of magnitude smaller than the Super-Kamiokande bound on λ. The difference in magnitudes is so great that there is clearly no prospect of confronting the prediction of Eq. (10) in the Super-Kamiokande experiment. The discrepancy between this conclusion, and the much more optimistic one reached by Lisi et. al., arises as follows. Lisi et. al. assume, on the basis of the general form for the decoherence term given in Eq. (1), the estimate

$$\mathcal{D} \sim H^2/M_{\rm Planck} \quad , \tag{11}$$

with H a characteristic energy (such as the neutrino energy) of the system. However, once the decoherence term is restricted to have the self-adjoint Lindblad form of Eq. (3), which is explicitly assumed in the analysis of Lisi et. al., the double commutator structure implies that the estimate is changed to

$$\mathcal{D} \sim (\Delta H)^2/M_{\rm Planck} \quad , \tag{12}$$

with ΔH the *energy variance*. [Note that the estimate of Eq. (12) is manifestly independent of the zero point with respect to which energies H are measured, whereas the estimate of Eq. (11) is not.) In a two level system, $\Delta H = |E_1 - E_2|$, and if the energy differences arise from mass differences in the two beam components, we evidently have $\Delta H \simeq \Delta m^2/(2E) = k$. This gives the estimate

$$\mathcal{D} \sim \frac{(\Delta m^2)^2}{4E^2 M_{\rm Planck}} \quad , \tag{13}$$

which because of the small neutrino mass difference is much more pessimistic than that of Eq. (11). Analogous remarks apply to tests for gravitation induced decoherence effects in the K and B meson systems, with Δm^2 replaced by the appropriate squared mass difference.

Although we have focused our discussion on the case of energy driven dissipation, because this corresponds to the analysis of Lisi et. al., if we assume instead the spontaneous localization model the estimates for the Super-Kamiokande experiment are equally pessimistic. In the spontaneous localization model, D in Eq. (8) is taken (in the small separation approximation for single particle dynamics) as the coordinate operator q, and the parameter σ^2 is given by $\sigma^2 = 2\lambda\alpha$, with $\lambda = 10^{-16}\mathrm{sec}^{-1}$ the localization frequency, and with $\alpha^{-\frac{1}{2}} = 10^{-5}\mathrm{cm}$ the localization distance. Thus in a two-level system, an estimate of the dissipative term $\mathcal{D}$ is

$$\mathcal{D} \sim 10^{-6}\mathrm{sec}^{-1}\mathrm{cm}^{-2}(\Delta q)^2 \quad ; \tag{14}$$

estimating $\Delta q = |q_1 - q_2|$ as the separation between centers of the ν_μ and ν_τ wave packets resulting from their mass difference, we get

$$\Delta q \sim \frac{1}{2}\Delta m^2 L E^{-2} \quad , \tag{15}$$

with L the neutrino flight path. To get an upper bound, we use the smallest Super-Kamiokande neutrino energy $E \sim 10^{-1}\mathrm{GeV}$ and the largest flight path $L \sim 10^4\mathrm{km}$, giving

$$\Delta q < 10^{-10}\mathrm{cm} \quad . \tag{16}$$

When substituted into Eq. (14) this gives the estimate

$$\mathcal{D} < 10^{-26}\mathrm{sec}^{-1} \sim 10^{-50}\mathrm{GeV} \quad , \tag{17}$$

again much smaller than the limit $\sim 10^{-23}$GeV placed on the decoherence term by the Super-Kamiokande experiment.

To conclude, we see that in the spontaneous localization model for the decoherence term, as well as in the energy conserving model, the predicted effect for the Super-Kamiokande experiment is proportional to the square of the neutrino mass squared difference Δm^2, and hence on the basis of these models is unobservably small. One could get an observable Super-Kamiokande effect within the framework of these models only by positing a much larger coefficient for the decoherence term than has been generally assumed in the state vector reduction context; it would be interesting to see if this has testable consequences elsewhere.

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[9] See, e.g., D. I. Fivel, quant-ph/9710042; L. P. Hughston, Ref. [8], and further references cited therein.